CONVECTIVE HEAT AND MASS TRANSFER IN A
MOVING ORDERED SYSTEM OF DROPS (BUBBLES)
FOR HIGH PECLET NUMBERS

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The structure of the singular streamlines, starting and terminating on the drop surfaces [1-3], plays a substantial part in problems on the convective diffusion in a system of reacting drops (bubbles) at high Péclet numbers. It here turns out that chains of drops exist in the flow, in which the internal mass transfer is frozen strongly by the interaction between the diffusion tracks and the boundary layers of the drops belonging to the chain.

1. Let us examine the process of convective diffusion in a concentrated system of spherical drops being precipitated or of bubbles being splashed, of radius a which move in an infinite volume of a fixed fluid. Let us consider the flow around an individual particle of the system to be laminar, and the velocity U of the motion of all the drops (bubbles) to be identical. During their motion let the drops form rectilinear chains, where the spacing between the drops in each chain is constant and equal to b, and the spacing between adjacent chains is the same in order of magnitude as the spacing between particles in one chain.

The situation described is realized in practice in the accomplishment of extraction and bubbling processes, for example. In the first case the assumptions made can be considered valid when drop insertion in the extraction column is accomplished at the same points at equal time intervals, and in the second, for a constant discharge of the bubbling gas, which permits an approximate consideration of the size of the bubbles being formed and the distance between them to be identical in each chain. The model to be examined below can be considered a rough mathematical model of mass transfer in extraction and bubbling processes.

The stream function near the drop (bubble) surface can be represented in the form

$$\psi = UB_\alpha(n)a(r - a) \sin^2 \theta,$$

in a spherical coordinate system coupled to the center of an arbitrary drop, where n is the number of drops (bubbles) per unit volume. The specific expression for $B_\alpha(n)$ can be determined within the framework of the cellular model, for instance (see [4] for small Reynolds numbers) or by using the point-forces model [5]. In particular, we have for $a/b \ll 1$

$$B_1 = (1/2)(\beta + 1)^{-1}, B_2 = 3/2,$$

where B_1 and B_2 correspond to the Stokes mode of flow around the drops (the Reynolds number is Re = $aU/\nu \ll 1$, where ν is the kinematic viscosity of the surrounding fluid) and to potential flow (Re $\gg$ 1), respectively, and β is the ratio between the viscosities of the drop and of the fluid surrounding it.

The convective diffusion process can be considered quasistationary in a coordinate system coupled to a moving system of drops (bubbles). This last assumption is satisfied approximately if the change in reagent concentration within the drop can be neglected, as is valid, e.g., during a rapid chemical reaction in the drops or for high reagent concentrations within the drop (bubble). Let us consider the Peclet number of an individual drop to be Pe = $aU/D \gg 1$ (D is the diffusion coefficient). We define the location of a fixed drop in the chain by its number k, where the numbering starts with the drop going in advance.

The concentration distribution in the flow is determined by the solution of the stationary convective diffusion equation

$$(\mathbf{v}\nabla)c = D\Delta c$$

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with boundary conditions of constancy of the concentration far from the system and constancy of the concentration of the substance dissolved in the flow, on the drop surfaces.

For high Peclet numbers, the main change in concentration will occur in the thin diffusion boundary layer of each drop in which the tangential transfer of material along the drop surface can be neglected as compared with the radial transfer, and also in the domain of diffusion traces located in the neighborhood of the singular streamlines starting and terminating on the particle surfaces. Hence, the diffusion boundary–layer equation with a leakage condition which depends on the relative location of the drop in the chain and is given by the concentration distribution in the diffusion layer of a drop located upstream [1-3], must be solved to determine the concentration near a fixed drop.

We consider below that the period of the chain satisfies the condition $b/a \ll Pe^{1/2}$. Hence, the leakage condition for the k-th sphere of the chain is determined by the concentration distribution in the convective boundary-layer region of the diffusion trace of the preceding $(k - 1)$-th drop.

The convective boundary layer region is characterized by the fact that the concentration therein remains constant on the streamlines and is determined by the concentration at the output from the diffusion boundary layer. This permits reduction of the initial problem to a problem of mass transfer of chains of drops and utilization of results [1-3] which result in expressions for the total diffusion fluxes on the drop surface under the assumption that the undepleted solution has a concentration c:

$$I_k = I_1[k^{1/2} - (k - 1)^{1/2}],$$
$$I_1 = 2^3 3^{-1/2} \pi^{1/2} B_\alpha^{1/2}(n) a^{3/2} U^{1/2} D^{1/2} c. \tag{1.1}$$

The concentration is here measured from its value on the drop surface.

Taking account of (1.1), we obtain for the mean diffusion flux on the drop

$$\langle I \rangle = k^{-1} \sum_{i=1}^{k} I_i = I_1 k^{-1/2}. \tag{1.2}$$

We now consider the number of drops in the system to be large, i.e., $k \to \infty$, and we determine the distribution of the mean concentration (the concentration outside the diffusion traces and boundary layers is later called the concentration in the core of the flow) along the stream axis.

Since the concentration in the core of the flow will vary slowly in spacings on the order of the period of the chain, a representative volume which substantially changes the scale in the variation of the concentration but contains a large volume of drops can be introduced.

Let us introduce the slow coordinate x measured along the flow. At the centers of the drops (bubbles) it takes on the value

$$x = x(k) = kb. \tag{1.3}$$

Taking account of (1.3) and the equation for the concentration in the core of the flow (c_0 is the undepleted concentration at the entrance to the layer of drops),

$$-U \partial c / \partial x = n \langle I \rangle, \ x = 0, \ c = c_0, \tag{1.4}$$

we obtain from (1.2) the distribution of the mean concentration along the flow

$$c = c_0 \exp \{-2\sigma U^{-1} x^{1/2}\},$$
$$\sigma = 2^3 3^{-1/2} \pi^{1/2} n b^{1/2} B_\alpha^{1/2}(n) a^{3/2} U^{1/2} D^{1/2} c. \tag{1.5}$$

Let us note that the concentration distribution obtained is applicable for both large and small Reynolds numbers of the laminar streamline of an individual drop (bubble).

The results obtained permit determination of the time dependence of the mean reagent concentration in the continuous phase at any point of a reactor z (here z is the spacing from the entrance to the reactor (see Fig. 1), under the assumptions made, in the form

$$c = c_0 \exp \{-2\sigma U^{-1}(Ut - z)^{1/2}\}.$$

z

$x=0(z=Ut)$ $c=c_0$
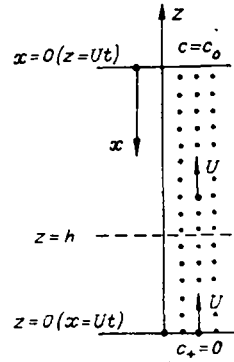
x

U

$z=h$

U

$z=0(x=Ut)$ $c_+=0$

Fig. 1

It is seen from (1.5) that at spacings on the order of the lattice period, the concentration in the core of the flow does not vary in practice: $c(x + b) \approx c(x)$. This property is destroyed near the point $x = 0$ since $\lim_{x\to 0} \partial c/\partial x = \infty$, and this is natural because it was assumed in the introduction of the slow coordinate x (1.3) that the number of drops per unit volume is large $(k \to \infty)$, and this, in turn governs the boundary of applicability of the formulas obtained in the coordinate x in the form of the inequality $x \gg b$ (see (1.3)).

2. Using the results of Sec. 1, we consider the problem of mass transfer between a moving ordered system of drops (bubbles) and a fixed fluid within the framework of the following, most simple model of two mutually penetrating continuous media between which mass transfer of continuous and discrete phases occurs.

We consider the discrete phase to move upward at a constant velocity U and at a distance x from its upper boundary to have a concentration $c_+(x, t)$ at the time t, while the continuous phase is fixed and has the concentration $c(x, t)$ (see Fig. 1).

The mass conservation law for each phase has the following form in a coordinate system coupled to the discrete phase

$$\partial c_+/\partial t = -dc/dt = J(t, x, c - c_+), \tag{2.1}$$

where $d/dt = \partial/\partial t + U\partial/\partial x$ is the total derivative and J is the mean quantity of reacting material per unit volume per unit time.

A specific expression can be obtained for J by using (1.2)–(1.4) for the mean diffusion flux on a drop per unit volume and by taking into account that the corresponding quantities were measured from the appropriate value of the concentration in the discrete phase

$$J = n\langle I\rangle = \sigma x^{-1/2}(c - c_+). \tag{2.2}$$

The boundary conditions of nondepletion of the concentration in the continuous phase on the outer boundary of the region for $x = 0$, and zero discrete phase concentration at the entrance to the reactor for $x = Ut$ must be added to the system of equations (2.1) and (2.2).

System (2.1) and (2.2) becomes in the variables $z = Ut - x$, x

$$L_1(c, c_+) = U\partial c/\partial x + \sigma x^{-1/2}(c - c_+) = 0,$$
$$L_2(c, c_+) = U\partial c_+/\partial z - \sigma x^{-1/2}(c - c_+) = 0, \tag{2.3}$$
$$x = 0, c = c_0, z = 0, c_+ = 0.$$

System (2.3) is linear and has a singularity at $x = 0$, completely analogous to that mentioned in Sec. 1. Without performing an exact investigation of the solution of system (2.3), which can be obtained by applying the Laplace transform in the coordinate z, let us just indicate its approximate expression for large values of x. This case corresponds to a concentration distribution in the phases at a fixed distance $z \le h = const$ from the entrance to the reactor for large time t (see Fig. 1). The appropriate expressions for the concentration have the following form in the t, z variables $(t \to \infty)$

$$c(t, z) = c_0 \exp\{-2\sigma U^{-1}(Ut - z)^{1/2}\},$$
$$c_+(t, z) = c(t, z)(1 - \exp\{-\sigma U^{-1}z(Ut - z)^{-1/2}\}). \tag{2.4}$$

For any t and z, expressions (2.4) satisfy the last equation of (2.3) and all the boundary conditions exactly, while the first equation of (2.3) is satisfied approximately since as $t \to \infty$ and $z \leq h$

$$\mathbf{L}_1(c_1, c_2) \leqslant c_0 \sigma (Ut - h)^{-1/2} \exp\{-2\sigma U^{-1}(Ut - h)^{1/2}\} \to 0.$$

It is seen from (2.4) that the reagent concentrations in the phases near the reactor entrance equalize with the lapse of time, reaching the value $c \approx c_+ \approx 0$.

Let us note that the approximate expression for the concentration $c(x, t)$ in the continuous phase (2.4) yields the exact value at the reactor entrance for $z = 0$. This is proved by direct integration of the first equation in (2.3) for $z = 0$ with the equality $c_+(t, 0) = 0$ taken into account.

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LASER CAVITATION IN LIQUID NITROGEN

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Since the first discovery of laser cavitation in liquids [1] a large number of studies of this phenomenon have appeared. The problem is of interest because this is in practice the only way of producing an isolated cavitation bubble within a liquid (with electrical discharges distortions are produced by the presence of the electrodes) and also because of the uncertainty surrounding the state of the material realized when such a cavity collapses. Studies have been made of the dynamics of bubbles formed by laser breakdown in a liquid using a technique based on recording of acoustical and light impulses produced during bubble formation and collapse [1], with high-speed photography [2], and by the shadow method with background illumination by a gas laser [3].

The goal of the present study is to investigate the laser cavitation in a most-simple cryogenic liquid — liquid nitrogen. Due to the closeness of the liquid nitrogen temperature to the boiling point, the pressure within the cavitation cavity at the latter's maximum dimensions, determined basically by the saturated nitrogen vapor pressure, will differ only insignificantly from the external pressure and the degree of bubble compression R/r will be small (here R is the maximum; r, the minimum bubble radius, respectively).

The temperature T within the bubble at maximum bubble compression can be written in the adiabatic approximation as

$$T = T_0(R/r)^{3(\gamma - 1)},$$